

July, 2006

Lower and Upper Bounds on CSL Parameters from Latent Image Formation and IGM Heating

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arXiv:quant-ph/0605072v9 24 Sep 2007

ABSTRACT

We study lower and upper bounds on the parameters for stochastic state vector reduction, focusing on the mass-proportional continuous spontaneous localization (CSL) model. We show that the assumption that the state vector is reduced when a latent image is formed, in photography or etched track detection, requires a CSL reduction rate parameter λ that is larger than conventionally assumed by a factor of roughly $2 \times 10^{9\pm 2}$, for a correlation length r_C of 10^{-5}cm . We reanalyze existing upper bounds on the reduction rate and conclude that all are compatible with such an increase in λ . The best bounds that we have obtained come from a consideration of heating of the intergalactic medium (IGM), which shows that λ can be at most $\sim 10^{8\pm 1}$ times as large as the standard CSL value, again for $r_C = 10^{-5}\text{cm}$. (For both the lower and upper bounds, quoted errors are not purely statistical errors, but rather are estimates reflecting modeling uncertainties.) We discuss modifications in our analysis corresponding to a larger value of r_C . With a substantially enlarged rate parameter, CSL effects may be within range of experimental detection (or refutation) with current technologies.

1. Introduction

Stochastic modifications of the Schrödinger equation have been intensively studied as models for objective state vector reduction [1]. As currently formulated, the reduction rate parameters for these models are many decades smaller than current experimental bounds, and will not be detectable in planned nanomechanical and gravitational wave detector experiments [2]. In order to motivate further experimental searches for stochastic modifications of Schrödinger dynamics, it is important to have lower bounds on the stochastic model parameters, below which one can assert that known measurement processes will not occur. Setting new, more stringent lower bounds, while at the same time reanalyzing and improving upper bounds, is the aim of this paper.

Conventional lower bounds on the stochastic parameters are based on an idealized measurement model in which the experimenter reads out results from the position of a macroscopic pointer. In such measurements, the detection and amplification processes needed to get a pointer readout are necessarily linked. Our focus in this paper is on a different type of experiment, in which first a latent image is formed, either in a photographic emulsion or a solid state track detector. Only long after latent image formation is amplification brought into play, in the form of development of the photographic plate, or etching of the track detector. A qualitative discussion of latent photographic image formation was given in 1993 by Gisin and Percival [3], who consider detection to have occurred already at the microscopic level, *before* the amplification associated with development. To quote them, “Of particular importance is the study of the formation of the latent image in photography, for this is not only the most common quantum detection technology, but it also shows unequivocally that amplification up to the macroscopic level is quite unnecessary for the formation of a

permanent classical record of a quantum event, by contrast with the example of the pointer, which is so often used.” In this paper we shall put the discussion of Gisin and Percival on a quantitative footing, within the framework of the continuous spontaneous localization (CSL) model with mass-proportional couplings.

This paper is organized as follows. In Sec. 2 we give a quick review of the CSL model with mass-proportional couplings, and in particular give rate formulas needed for the subsequent discussion. In Sec. 3 we discuss the formation of latent photographic images in the Mott-Gurney model, and show that with standard parameter values, the CSL model predicts a reduction rate that is a factor of order $\sim 2 \times 10^{9 \pm 2}$ too slow, as compared with the estimated rate of latent image formation. In Sec. 4, we make analogous (but cruder) estimates for solid state etched track detectors, and again conclude that the usual CSL parameter values cannot account for latent image formation.

In Sec. 5, we examine whether various upper bounds on the CSL parameters allow a substantial enlargement in the reduction rate parameter. Processes considered include Fullerene diffraction, supercurrent persistence, proton decay, spontaneous radiation from germanium, cosmic IGM heating effects, and planetary heating. Our conclusion in all cases not involving heating (in some cases disagreeing with previous analyses) is that an increase of λ by a factor of 10^{12} is allowed by experimental data. Heating of the IGM places a stronger constraint on λ , allowing an increase by a factor of $\sim 10^{8 \pm 1}$ over the standard value. Although planetary heating nominally places a much more stringent bound, we argue that competition with molecular collision effects, which are 28 orders of magnitude larger and are strongly dissipative, invalidates this bound.

In Sec. 6, we discuss modifications in our analysis resulting from changing the cor-

relation function $g(x)$, and from changing the value of the correlation length r_C from the standard value 10^{-5} cm assumed in CSL analyses. In Sec. 7 we discuss implications of our suggested new values for the CSL parameters for experiments to directly test for CSL effects, focusing on large molecule diffraction, superconductor current decay, nanomechanical and gravitational wave detection experiments, and the Collett-Pearle [4] proposal to observe rotational Brownian diffusion. We also briefly consider the competition of molecular collisions with CSL effects. Finally, in Sec. 8 we discuss and summarize our results, and in particular, we note that our key assumption, that latent image formation already constitutes measurement, is subject to direct experimental test.

2. The mass-proportional CSL model

We begin by stating some standard formulas of the mass-proportional CSL model, drawing heavily on the review of Bassi and Ghirardi [1]. The basic stochastic differential equation of the model is

$$d|\psi(t)\rangle = \left[-\frac{i}{\hbar} H dt + \int d^3x (M(x) - \langle M(x) \rangle) dB(x) - \frac{\gamma}{2} \int d^3x (M(x) - \langle M(x) \rangle)^2 dt \right] |\psi(t)\rangle , \quad (1)$$

with $dB(x)$ a Brownian motion obeying

$$dt dB(x) = 0 , \quad dB(x) dB(y) = \gamma \delta^3(x - y) dt , \quad (2)$$

with $\langle M \rangle$ denoting the expectation of M in the state $|\psi(t)\rangle$, and with the operator $M(x)$ given by

$$M(x) = m_N^{-1} \int d^3y g(x - y) \sum_s m_s N_s(y) . \quad (3)$$

In Eq. (3) the sum extends over particle species s of mass m_s and with number density operator $N_s(y)$, while m_N is the mass of the nucleon and $g(x)$ is a spatial correlation function conventionally chosen as

$$g(x) = \left(\frac{\alpha}{2\pi}\right)^{3/2} e^{-(\alpha/2)x^2}, \quad \int d^3x g(x) = 1 \quad . \quad (4)$$

(In Sec. 6 we will show that changing the functional form of the correlation function does not substantially alter our conclusions.) This correlation function can be interpreted as the functional “square root” of the correlation function for a noise variable that couples locally to the mass density $\sum_s m_s N_s(y)$. That is, writing $dC(y) = \int d^3x g(x-y) dB(x)$, which has the spatial correlation function $dC(y)dC(z) = \gamma dt \int d^3x g(x-y)g(x-z)$, the noise term in Eq. (1) takes the locally coupled form $\int d^3y dC(y) m_N^{-1} \sum_s m_s (N_s(y) - \langle N_s(y) \rangle)$. Thus, the basic parameters of the model are the strength of the Brownian process γ , and the correlation function width parameter α .

It is convenient to introduce two further parameters that are defined in terms of γ and α . Since α has the dimensions of inverse squared length, we define a correlation length r_C (sometimes denoted by a in the CSL literature) by

$$r_C = (\alpha)^{-1/2} \quad , \quad (5a)$$

which is conventionally assumed to take the value of $r_C = 10^{-5}$ cm. Additionally, we introduce a rate parameter λ defined by

$$\lambda = \gamma \left(\frac{\alpha}{4\pi}\right)^{3/2} = \gamma / (8\pi^{3/2} r_C^3) \quad , \quad (5b)$$

which is conventionally assumed to take the value $\lambda = 2.2 \times 10^{-17} \text{ s}^{-1}$, giving γ the value $10^{-30} \text{ cm}^3 \text{ s}^{-1}$.

We can now state two key rate formulas that we will need for the subsequent analysis. According to Eq. (8.15) of Bassi and Ghirardi [1], the off-diagonal coordinate space density matrix element $\langle \ell | \rho | 0 \rangle$ for a single nucleon approaches zero exponentially with a reduction rate Γ_R given by

$$\Gamma_R = \lambda \left(1 - e^{-\ell^2 / (4r_C^2)} \right) \quad , \quad (6a)$$

which for ℓ comparable to or larger than r_C , can be approximated as

$$\Gamma_R \simeq \lambda \quad . \quad (6b)$$

For n nucleons within a radius smaller than the correlation length, this rate is multiplied by n^2 ; for N groups of nucleons separated by more than the correlation length, this rate is multiplied by N , and for particles of mass m_p the rate is multiplied by $(m_p/m_N)^2$, giving

$$\Gamma_R \simeq \lambda n^2 N (m_p/m_N)^2 \quad . \quad (6c)$$

The formulas of Eqs. (6a-c) will be the basis of our reduction time estimates for latent image formation. Before proceeding further, we note that there are other natural definitions of a reduction rate; for example, if it is defined by the rate of approach to zero of the variance in the coordinate, as in Adler [1], rather than the rate of vanishing of the off-diagonal density matrix element, then Γ_R is double that given by Eqs. (6a-c).

A second important formula gives the rate of secular center-of-mass energy gain, as a result of the Brownian process, for a body comprised of a group of particles of total mass M . This is given by the formula (Bassi and Ghirardi [1], Pearle and Squires [5], Adler [6])

$$\frac{dE}{dt} = \frac{3}{4} \lambda \frac{\hbar^2}{r_C^2} \frac{M}{m_N^2} \quad . \quad (7)$$

This formula will be used to set upper bounds on the reduction rate parameter λ .

3. Latent image formation in photography

The case of latent image formation in photography is of particular interest because the photographic process has been intensively studied, both theoretically and experimentally. Survey accounts of what is known are given in the books of Mott and Gurney [7] and Avan et al [8], and in the review articles of Berg [9] and of Hamilton and Urbach [10].

A photographic emulsion consists of grains of silver halide suspended in gelatine. The typical grain size is between 1/10 of a micron and a few microns, or in other words, from 10^{-5} cm to a few times 10^{-4} cm in diameter. Grains are typically spaced around 1 to 2.5 microns apart in the gelatine. The basic microscopic theory originated by Gurney and Mott [11] is illustrated graphically on p 75 of ref [8], for a typical AgBr emulsion. The steps as envisaged by Gurney and Mott are as follows. First a photon is absorbed in the grain, giving rise to an electron and a hole. The electron gets trapped on the surface of the grain, and the hole produces a neutral Br which is absorbed on the surface. An interstitial ion of Ag^+ then diffuses to the trapped electron and forms a neutral silver atom. A second photon is absorbed, giving rise to another electron and hole; the hole produces a second neutral Br which joins with the first to produce a Br_2 molecule, which eventually diffuses out of the grain into the gelatine, while the electron combines with the neutral Ag on the surface to give an ion Ag^- . (As shown in Fig. 5.4 of Hamilton and Urbach [10], the bromine that has diffused into the gelatine typically moves up to around a micron away from the grain; this will figure in our discussion of Sec. 6.) Finally, a second interstitial Ag^+ diffuses to the surface to join the Ag^- , forming a molecule Ag_2 . This process is then repeated until a cluster of free silver atoms (a silver speck) has been formed on the surface of the grain, making it developable. As discussed in detail in ref [8], an alternative microscopic model has

been proposed by Mitchell, in which crystalline imperfections play an important role, and in which the order of steps differs from that in the theory of Gurney and Mott. However, such details are not relevant to the estimates that we shall make, which only use the number of ions that move and the distance that they travel, but not the order in which the motions take place, details of trapping, etc.

There seems to be general agreement that around 3 to 6 silver atoms are needed to give a grain a 50% probability of being developable, and about 30 silver atoms are needed for a grain to be certain to be developable. In using grains to define a particle track in a 100 micron width emulsion, a few grains developable by random processes are encountered, so we shall assume that a minimum number of about $N = 20$ developable grains is required to define a track. Assuming that 30 Ag and Br atoms (with respective atomic weights of 108 and 80) move a distance greater than or of order r_C in the ionic step of latent image formation, we have for the number of nucleons that move $n = 30(108 + 80) = 5640$. So from Eq. (6c), we have for the reduction rate Γ_R for the process of latent image formation producing the track,

$$\Gamma_R = \lambda n^2 N = 2.2 \times 10^{-17} \text{s}^{-1} \times 5640^2 \times 20 = 1.3 \times 10^{-8} \text{s}^{-1} \quad . \quad (8)$$

On the other hand, Mott and Gurney [7] estimate in their Table 28 the rate of growth of a silver speck. Scaling their room temperature example to a single speck of radius $0.2 \times 10^{-5} \text{cm}$, which fits comfortably within a grain of diameter 10^{-5}cm , one finds a rate for accumulating 30 silver atoms of around 30s^{-1} . This rate should be considered uncertain by two orders of magnitude in either direction; for example, using the ion step time of $1/25,000 \text{s}$ discussed on p 248 of Mott and Gurney, the rate to accumulate 30 silver atoms would be about 1000s^{-1} , while applying the results of Table 28 to a speck of radius $0.2 \times 10^{-7} \text{cm}$,

corresponding to just a few silver atoms, would give a rate to accumulate 30 atoms of 0.3s^{-1} . Note that since the accumulation of silver atoms on a speck is a sequential process, the time for building up the speck can be much longer than the duration of the exposure that initially produces electrons and holes in the interior of the grain.

If we identify the time to form a developable speck with the total time to form the latent image, we see that a reduction rate of $3 \times 10^{1\pm 2}\text{s}^{-1}$ is required for reduction to be completed during latent image formation. Thus, based on this estimate, the rate of Eq. (8) is too small by a factor of $2 \times 10^{9\pm 2}$; in other words, the CSL rate parameter λ would have to be increased by this factor to account for latent image formation (for r_C fixed at 10^{-5}cm).

Let us now address a possible objection to this conclusion, that we have considered only the mass motion associated with registration of the latent image, but have neglected possible motions of environmental particles induced by the registration process. The first thing to be said is that photographic emulsions work perfectly well in vacuo, so it suffices to consider just the motions of particles within the emulsion. When a photon is absorbed by an atom, the atom will recoil, potentially altering the phonon distribution, and thus displacing all the atoms in the detector. For a photon of energy a few eV absorbed by a silver halide molecule, this recoil will have an energy E_R of order 10^{-10}eV , which is much less than the typical Debye temperature energy $E_D \sim 10^{-2}\text{eV}$. Thus the parameter E_R/E_D governing the probability of a phonon-free absorption is of order 10^{-8} , and so the recoil will be absorbed, with probability $1 - \text{O}(10^{-8})$, by a translation of the whole detector, without the emission of phonons. Hence for relevant elapsed times t , the recoil distance ℓ will be much smaller than the correlation length, and we can use the expansion of Eq. (6a) given in Eq. (9) below. Given a total detector mass of Nnm_N , the recoil distance will be $\ell = pt/(Nnm_N)$, with p

the incident photon momentum, and so the reduction rate associated with detector recoil will be $\Gamma_R = [\lambda/(4N)][pt/(m_N r_C)]^2$. Notice that the factor n^2 has canceled out, and the rate is proportional to the inverse of the emulsion size, as reflected in the factor N^{-1} . For a 3eV incident photon, a correlation length $r_C = 10^{-5}\text{cm}$, and an elapsed time t of 1/30s, this estimate gives $\Gamma_R \sim 0.5N^{-1} \times 10^{-6}\text{s}^{-1}$ for the standard CSL value of λ . For a minimal detector with $N = 20$ we obtain a reduction rate about a factor of two larger than that estimated in Eq. (8), but for a one cubic centimeter emulsion, with $\sim 10^{24}$ nucleons, the effective N in the recoil estimate is 10^{15} , and the recoil induced reduction rate is negligible.

Once the incident photon has been absorbed, the remaining steps in latent image formation involve transport and diffusion of electrons, holes, or silver and bromine ions. The electron liberated from a silver halide molecule by the absorbed photon will initially have a velocity larger than the mean thermal velocity, and will emit phonons until it slows down to thermal velocity. This effect is calculated in Appendix D, where we estimate that it leads to a reduction rate $\Gamma_R \sim 10^{-17}\text{s}^{-1}$, nine orders of magnitude smaller than the result of Eq. (8). Assuming that interstitial ions are already thermalized, and that they have a dispersion relation of the form $E = p^2/2m^*$ with m^* an effective mass, the corresponding phonon emission reduction rates due to ionic motions are much smaller, because only the tail of the ionic thermal momentum distribution is kinematically allowed to emit phonons. Once thermal equilibrium is established, only the difference between the thermal motions in superposed states in the wave function can contribute to reduction. This difference should be smaller than twice the thermal motion in one of the states of the superposition, and an estimate given in Appendix D shows that this bound on the thermal contribution to the reduction rate is similar in magnitude to the result of Eq. (8). Finally, the motion of a

silver (or bromine) ion from the interior of a grain to the surface requires a compensating back motion of the whole emulsion if no momentum is transferred to the surroundings. For example, if a silver ion diffuses a distance r_C , to keep the center of mass of the emulsion fixed, it must translate by $108r_C/(Nn)$, leading to a reduction rate $\Gamma_R \sim N^{-1} \times 10^{-13}\text{s}^{-1}$ for the standard CSL value of λ . This is negligibly small even for $N = 1$, that is, for an emulsion consisting of a single grain. In sum, these estimates suggest that collective motions of the atoms in the emulsion, induced by the process of latent image formation, do not alter our conclusion that the standard CSL value of λ is not large enough to account for latent image formation.

4. Latent image formation in etched track detectors

In this section we turn to the consideration of a second type of latent image formation, in which etchable defects are formed in solid state detectors. Using related parameters, we will also estimate the reduction time associated with electronic motions resulting from passage of the charged particle through the detector.

Detailed accounts of etched track detectors are given in the books of Durrani and Bull [12] and Avan et al [8], from which we draw the needed parameters. Passage of a charged particle results in a cylindrical region of lattice distortion around the charged particle path, with a typical radius of 6 to 12 nm (i.e., 60 to 120 Å, or 0.6 to 1.2×10^{-6} cm), which we take as giving the displacement ℓ in Eq. (6a). Since this ℓ is considerably smaller than r_C , we expand the exponential to get the associated reduction rate

$$\Gamma_R = \lambda n^2 N \ell^2 / (4r_C^2) \quad , \quad (9)$$

where as before, n is the number of nucleons within a correlation length, and N is the number

of correlation-length groups of nucleons in the particle track. Let us estimate the length of a short ionic track length to be 10 microns, giving $N = 100$. For n , we take the number of nucleons in a cylinder of diameter $\ell = 10^{-6}$ cm, and length 10^{-5} cm, assuming a density of 10^{24} nucleons per cubic centimeter. This estimate gives $n \simeq 3 \times 10^7$. Thus, with the standard value $\lambda = 2.2 \times 10^{-17} \text{s}^{-1}$, we get a reduction rate of $\Gamma_R \simeq 7 \times 10^{-3} \text{s}^{-1}$. Various numbers for the time t for a track to form are given in the two books cited. A minimum for t is given by the lattice vibration time of 10^{-13}s ; according to Table VIII.3 of Avan et al, establishment of thermal equilibrium requires a time in the range of 10^{-12}s to 10^{-9}s , and establishing chemical equilibrium requires a time of a few times 10^{-8}s to much longer, depending on the material. Thus, these time estimates indicate that the standard value of λ gives a reduction rate that is too small by a factor of 10^{11} to 10^{14} , if thermal equilibrium is taken as the criterion for latent image formation, and a factor of around 5×10^9 or less, if chemical equilibrium is used as the criterion.

Let us next address a potential objection to this conclusion, and to the similar one reached in Sec. 3 in the case of photographic emulsion grains, that we have neglected the mass transport associated with the ionization produced by passage of the charged particle. However, the initial ionization consists almost entirely of the motion of electrons, and according to Katz and Kobetich [13], the bulk of the energy deposited within 20\AA of the track comes from δ -rays with energy less than 0.1keV . As one might expect, these electrons have ranges of order 40\AA or less, so the geometry of electron displacements is essentially the same as the geometry of nucleonic displacements. Since the electron-associated reduction rate is smaller than that calculated above by a factor $(m_e/m_N)^2$, the electrons that are initially ionized in the course of track formation give a negligible correction to the estimate made

above.

5. Upper bounds on the CSL parameter

In the previous two sections, we have concluded that if state vector reduction occurs when a latent image is formed, the reduction rate parameter λ must be much larger than conventionally assumed. We turn now to an examination of whether such an enlarged λ is allowed by various experimental constraints. We continue to assume for this discussion that the correlation length r_C is 10^{-5}cm , an assumption to which we shall return in Sec. 6.

5A. Fullerene diffraction experiments

We begin our survey of experimental constraints with the fullerene diffraction experiments of Arndt et al [14], Nairz, Arndt and Zeilinger [15], and Nairz et al [16], which we previously discussed in Sec. 6.5 of Adler [1]. In these experiments, molecules with $n \sim 10^3$ nucleons are observed to produce interference fringes when diffracted through gratings with spacings of 1 to $2.5 \times 10^{-5}\text{cm}$, of order the correlation length r_C . From Eq. (6c), we get a reduction rate with the standard λ of $\Gamma_R = 2.2 \times 10^{-17}\text{s}^{-1} \times 1000^2 \simeq 2 \times 10^{-11}\text{s}^{-1}$. Since the beam transit time in these experiments is of order 10^{-2}s , the interference fringes will not be spoiled provided that $\Gamma_R < 10^2\text{s}^{-1}$, permitting the rate parameter λ to be up to 5×10^{12} times as large as the standard value.

5B. Decay of supercurrents

The decay of supercurrents in the mass-proportional CSL model has been calculated by Buffa, Nicosini and Rimini [17], as also reported in Rimini [18], extending to the CSL

case earlier work by Rae [19]. Equation (5.9) of ref [17] gives a fractional supercurrent decay rate, using the standard CSL parameters, of $5.1 \times 10^{-27} \text{s}^{-1}$. Since the current experimental limit is [20] about $10^{-5} \text{year}^{-1} = 3 \times 10^{-13} \text{s}^{-1}$, the rate parameter λ is permitted to be up to 10^{14} times as large as the standard value. (This limit will be increased when recombination processes leading to the formation of Cooper pairs are taken into account, since these are not included in the calculations of refs. [17], [18], and [19].)

The review of Bassi and Ghirardi [1] quotes in detail only the numbers obtained by Rae, although referring to the later work of refs [17] and [18]. Rae assumes a fractional decay rate given by λ , for which he takes the value 10^{-15}s^{-1} , as opposed to the CSL value of $2.2 \times 10^{-17} \text{s}^{-1}$. The CSL model calculation in refs [17] and [18] decreases the estimate given by λ by a factor of $(r_C k_F)^{-1} \simeq 0.6 \times 10^{-3}$ (with k_F the Fermi momentum) arising from indistinguishability of electrons, and a factor $(m_e/M_N)^2 \simeq 0.3 \times 10^{-6}$, arising from the reduced stochastic coupling of the electron in the mass-proportional scheme. Assuming that each spontaneous localization breaks a Cooper pair, and that Cooper pairs are not recreated, this gives an overall supercurrent decay rate of $2.2 \times 10^{-17} \text{s}^{-1} \times 0.2 \times 10^{-9} = 4.4 \times 10^{-27} \text{s}^{-1}$, essentially the same number given in ref [17].

The indistinguishability factor $(r_C k_F)^{-1}$ can be understood heuristically from the fact that localization within a radius r_C involves a momentum transfer $\delta k \sim 1/r_C$, and so is forbidden by the exclusion principle except for electrons within δk of the Fermi surface, which are a fraction of order $\delta k/k_F$ of all electrons. Since for temperatures well below the critical temperature all electrons in the Fermi sea contribute to the supercurrent, and not just those near the Fermi surface, spontaneous localization can thus affect only a fraction $(r_C k_F)^{-1}$ of the electrons in the supercurrent, and hence this factor appears in the supercurrent decay

rate.

5C. Excitation of Bound Atomic and Nuclear Systems

Pearle and Squires [5] have estimated the rate of bound state excitation in the mass-proportional CSL model and applied it to various physical systems. Their Eq. (12) gives for the rate \dot{P} of internal excitation of atoms

$$\dot{P} \sim \lambda(m_e/m_N)^2(a_0/r_C)^4 \quad , \quad (10)$$

with a_0 the atomic radius $\sim 10^{-8}$ cm, which with the standard value of λ gives $\dot{P} \sim 0.7 \times 10^{-35}\text{s}^{-1}$. Requiring that cosmic hydrogen not dissociate during the lifetime of the universe of 4×10^{17} s gives an upper bound on λ of 4×10^{17} times the standard value.

Also in ref [5], Pearle and Squires apply an analogous formula to proton decay, with a_0 now a nuclear radius of 10^{-13} cm, and with m_e replaced by the quark mass m_q . Using the constituent quark mass $m_q \sim m_N/3$, this gives $\dot{P} \sim 10^{-50}\text{s}^{-1}$, while using current quark masses of around 10 MeV gives $\dot{P} \sim 10^{-53}\text{s}^{-1}$. Requiring \dot{P} to be smaller than the proton decay rate, which is known to be less than $10^{-33}\text{year}^{-1} = 0.3 \times 10^{-40}\text{s}^{-1}$, allows λ to be up to $\sim 10^9$ times the standard value if constituent quark masses are used in the estimate, and up to $\sim 10^{13}$ times the standard value if current quark masses are used. However, these limits are overly restrictive because the formula of Pearle and Squires assumes that no selection rules are at work. This is true in the case of atomic dissociation, but not in the case of decay of a proton. Because free quarks are confined, and because fermion number is conserved modulo 2, allowed proton decay modes must contain at least one lepton, which is not originally present as a constituent of the proton. Hence the proton decay rate, which involves a wave function overlap squared, must be further suppressed by $(m_N/\Lambda)^4$, with Λ

the minimum scale at which physics beyond the standard model is found. Making the very conservative estimate $\Lambda \sim 250\text{GeV}$ (the electroweak scale) gives an additional suppression of 3×10^{-10} ; hence proton decay allows λ to be up to at least 10^{18} times the standard value, using the constituent quark mass estimate.

Yet another application of the excitation rate formula is given by Collett and Pearle [4], to an experiment in which the rate of spontaneous 11 keV photon emission from germanium, as monitored in 1 keV bins, has been bounded by 0.05 pulses/(keV kg day). The assumption here is that nuclear excitation will have a high probability of knocking out a 1s electron, with resulting emission of an 11keV photon as the remaining electrons cascade downward to fill the vacant orbit. To get an estimate we use Eq. (10) with m_e replaced by m_N , and a_0 taken as the nuclear radius given by $1.4 \times 10^{-13} A^{1/3}$ cm, with A the atomic number. Using $A \simeq 73$ for germanium, which has 8.3×10^{24} atoms/kg, and equating \dot{P} to the rate limit for the first 1keV bin, gives the bound $\lambda < 6 \times 10^{-3}\text{s}^{-1}$, which is $\sim 3 \times 10^{14}$ times as large as the standard CSL value of λ . (This bound is a factor of 10^3 higher than the one quoted by Collett and Pearle [4]; where they get $\lambda^{-1}r_C^4 > 2 \times 10^{-15}\text{cm}^4\text{s}$, our evaluation from their numbers gives $\lambda^{-1}r_C^4 > 2 \times 10^{-18}\text{cm}^4\text{s}$.) Yet another analysis of nuclear excitation has recently been given by Pearle [4], who used the CSL model to estimate the rate of quadrupole excitation of a germanium nucleus to its first excited state. Pearle shows that for this process, the experimental data on photon emission require λ to be less than $\sim 10^{14}$ times its standard value.

5D. Radiation by Free Electrons

Fu [21] has calculated the rate of radiation of photons of energy k by free electrons

that is induced by the stochastic term in the Schrödinger equation. Including the factor $(m_e/m_N)^2$ called for in the mass-proportional coupling case, his result becomes

$$\frac{d\Gamma(k)}{dk} = \frac{e^2 \lambda}{4\pi^2 r_C^2 m_N^2 k} \quad . \quad (11)$$

An alternative derivation of this result is given in Appendix A, obtained by calculating the mean squared acceleration produced by the stochastic term, and then using this in the standard formula for the power radiated by an accelerated charge. In evaluating his result numerically, Fu takes $e^2 = 1/137.04$, whereas the standard Feynman rules that he uses require the identification $e^2/(4\pi) = 1/137.04$. He also uses the value $\lambda = 10^{-16}\text{s}^{-1}$, so to correct for his evaluation of e^2 and to correspond to the CSL value of λ , we multiply the rate result of his Eq. (4.1) by $4\pi/4.5 = 2.8$. Comparing with the experimental bound on photon emission from germanium quoted above, this gives $\lambda < 1.7 \times 10^{-11}\text{s}^{-1}$, which is only $\sim 10^6$ times as large as the standard CSL value of λ .

However, in the special case of mass-proportional coupling that we are considering here, this bound is significantly modified. Although the valence electrons in germanium are quasi-free, they are still charge neutralized by the core consisting of the inner electrons and nucleus. The core has a center of mass motion with a characteristic lattice vibration period of order $2 \times 10^{-13}\text{s}$, much longer than the $4 \times 10^{-19}\text{s}$ period of an emitted 11 keV photon, so the core motion should give at most an order 10^{-6} correction to the rate of 11 kilovolt photon emission by the core induced by the stochastic term in the Schrödinger equation. In the mass-proportional coupling CSL model, since the core radius is much less than the correlation length, the core can be treated as a point particle with the stochastic term acting only on its center of mass. Moreover, the acceleration of the core induced by the stochastic term will be the same as that of electrons (the m^{-1} factor relating acceleration to force is

canceled by the m factor in the coupling to the noise), and so the core motion will tend to cancel the radiation field produced by the electron motions. The cancellation in the far-zone radiation field is incomplete because of two types of corrections. The first are order a_0/r_C corrections, reflecting the variation of the correlation function over the germanium atom radius $a_0 \sim 10^{-8}\text{cm}$, which gives a correction of order $(a_0/r_C)^2 \sim 10^{-6}$ times Fu's estimate in the radiated power. The second are retardation corrections of order v/c , with $v \sim 0.3 \times 10^{-3}c$ the thermal velocity of the valence electrons, which gives a correction of order $v^2/c^2 \sim 10^{-7}$ times Fu's estimate in the radiated power. (There will also be a cross-term correction in the radiated power of order $(a_0/r_C)(v/c) \sim 10^{-6}$ times Fu's value.) Thus, taking the effect of charge neutralization into account, Fu's calculation shows that λ can be at most $\sim 10^{12}$ times as large as the standard CSL value of λ .

5E. Heating of Protons

We consider next the heating of protons by the stochastic noise, as given by Eq. (7). Taking $M = m_N$, we get from Eq. (7) and the standard CSL parameters,

$$\frac{dE}{dt} = \frac{3}{4}\lambda \frac{\hbar^2}{r_C^2 m_N} = 6.8 \times 10^{-26} \text{eV s}^{-1} = 1.1 \times 10^{-37} \text{erg s}^{-1} \quad . \quad (12)$$

Hence over a ten billion year period ($3.15 \times 10^{17}\text{s}$), ignoring dissipation, a proton would gain an energy from this effect of $\sim 2 \times 10^{-8}$ eV, corresponding to a temperature of $\sim 2 \times 10^{-4}$ degrees Kelvin. If λ were larger than the standard CSL value by a factor of 2×10^7 , the minimum value of the range inferred from our latent image formation analysis, the temperature increase would be $\sim 7 \times 10^3$ degrees Kelvin over the age of the universe.

We use this number to get a number of different bounds. The first is obtained by supposing that if λ is much larger than the standard value, all of the energy gained by

protons in the universe is thermalized into low energy photons. The total energy per unit volume released this way must not exceed a small fraction, say 0.1, of the energy per unit volume in the 3 degree ($\sim 2.6 \times 10^{-4}$ eV) microwave background radiation. Taking account of the fact that there are $\sim 10^9$ microwave background radiation photons per proton in the universe, this gives the bound that λ must be less than $\sim 10^{12}$ of the standard CSL value.

A more precise way to get a cosmological bound from proton heating is to utilize the fact that there have been detailed experimental and theoretical studies of low density regions of intergalactic space containing the intergalactic medium (IGM) [22]. The IGM consists of highly ionized hydrogen, with a typical temperature of around 2×10^4 degrees Kelvin [22] as measured between redshifts of $z = 4$ and $z = 2$. The IGM is heated by radiation from various astrophysical sources (stars, supernovas, quasi-stellar objects) and is cooled by adiabatic expansion of the universe, and by recombination cooling of the plasma. We can get an upper bound on the proton heating rate, by ignoring possible astrophysical heating mechanisms, and assuming that all of the heating in fact comes from the CSL heating effect, which must be equated to the cooling rate rate from adiabatic expansion and recombination cooling to explain the observed temperature equilibrium. For the highly ionized IGM, recombination cooling is less important (by a factor of around 6 at $z = 3$) than adiabatic expansion, so to to get an estimate we shall ignore the influence of recombination cooling, and just consider the effect of adiabatic cooling as follows. Under adiabatic expansion, the temperature varies with redshift z as $T \propto (1+z)^2$, and so the thermal energy loss of a proton obeys $dE/dt = (d/dt)(3/2)kT = (3kT/(1+z))|dz/dt|$. At $z=3$ (for Hubble constant $H_0 = 71$ km/s/Mpc and for matter and dark energy fractions $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$) one has $|dt/dz| = 0.8 \times 10^9$ years. Assuming a temperature T at $z = 3$ of 2×10^4 degrees Kelvin, this gives an energy loss rate

of $0.5 \times 10^{-16} \text{eVs}^{-1}$. Equating this to an energy gain from proton stochastic heating, from Eq. (12), gives a value of λ equal to 8×10^8 times the standard CSL value as an upper bound.

A second IGM bound can be obtained from measurements at $z \simeq 0$, which are less precise than those in the range $2 \leq z \leq 4$. Ricotti, Gnedin, and Shull [22] give a $z = 0.06$ mean temperature of $\sim 10^{3.7 \pm 0.5}$ degrees Kelvin (with one sigma errors), which limits λ to be at most $10^{7.2 \pm 0.5}$ times as large as the standard CSL value. Combining the $z = 3$ IGM bound obtained in the preceding paragraph with this one, and rounding the error in the exponent to an integer, we get an overall IGM bound on λ of $\sim 10^{8 \pm 1}$ times the standard CSL value.

Yet another cosmological bound comes from considering interstellar dust grains. Here we compare the rate of energy accumulation, given by Eq. (12), with the rate of energy radiation by the grain. Assuming 10^{24} nucleons per cubic centimeter, the rate of volume energy production for the standard CSL value of λ is $\sim 7 \times 10^{-2} \text{eVs}^{-1} \text{cm}^{-3}$. Radiation from dust grains does not follow the Stefan-Boltzmann law (power \propto area $\times T^4$), but rather scales with the dust grain volume and the fifth power of the temperature [23], according to the following formula giving W , the rate of energy radiated per unit volume of grain,

$$W = 32\pi \, 24.9 \frac{c(kT_g)^5}{(hc)^4} \kappa' \quad . \quad (13)$$

Here h and c are the Planck constant and the velocity of light, T_g is the grain temperature, and κ' is the imaginary part of the refractive index. Taking typical values $T_g \sim 20$ degrees Kelvin and $\kappa' \sim 0.05$, this formula gives $W \simeq 2 \times 10^{14} \text{eVs}^{-1} \text{cm}^{-3}$. Hence the dust grain energy balance implies only a weak bound, that λ can be at most of order 10^{15} times its standard CSL value.

We finally consider bounds on energy production obtained from the energy balance associated with planetary heat flows. Table 6.3 of de Pater and Lissauer [24] gives the lumi-

nosity to mass ratio L/M for solar system objects. The lowest ratios in the table, $< 4, 4, 6.4$ in units of $10^{-8}\text{erg g}^{-1}\text{s}^{-1}$, come respectively from Uranus, carbonaceous chondrites, and Earth. Comparing with the nucleon heating rate of Eq. (12), which is $\sim 7 \times 10^{-14}\text{erg g}^{-1}\text{s}^{-1}$, and ignoring the fact that the nucleons in earth are not isolated, we would conclude that λ can be at most 5×10^5 times the standard CSL value. However, this bound is dubious, for the following reason. In a recent paper, Bassi, Ippoliti and Vacchini [25], following up on earlier work of Halliwell and Zoupas [26] and Gallis and Fleming [27] (see also Benatti et al [28]), point out that in stochastic models with a dissipative form, the rate of heating of nucleons is not constant, but approaches zero after a finite time, so that the energy gained through the heating effect approaches a finite limit as opposed to increasing indefinitely. Even when the fundamental stochastic reduction process is non-dissipative, for nucleons in the earth the reduction process is a much weaker effect (by about 28 orders of magnitude, for λ at the IGM upper limit) than ordinary collisional decoherence [29], which is dissipative. Hence any stochastic heating effect should have equilibrated to zero very rapidly as a result of the influence of molecular collision effects, and so Eq. (12) cannot be compared to the planetary heat flow to set a bound on λ .

6. Possible modifications in our analysis

We discuss in this section possible modifications in our analysis. We consider first a modification that might eliminate the upper bound obtained from IGM heating. The heating bound could be eliminated if, as discussed in ref [25], the stochastic Schrödinger equation itself were modified to include dissipative effects. If the energy gain by a proton from stochastic effects were to saturate, independent of the value of λ , at a level significantly

below 10^4 degrees Kelvin, then IGM heating would not lead to a bound on λ . However, as emphasized in ref [25], for a dissipative stochastic model to be a serious competitor to the standard CSL model, a way has to be found to give dissipative reduction a field theoretic formulation, so that Fermi and Bose statistics for identical particles can be properly taken into account, as is done in the CSL model. This is an important open problem needing further study, but in the absence of its solution we continue to assume that the stochastic Schrödinger equation is exactly nondissipative in form.

We consider next the effect on the heating bound of changing the correlation function $g(x)$ and the correlation length r_C . The first comment to be made is that changing the form of $g(x)$, as long as it remains a function only of the magnitude of x , has no qualitative effect on the analysis. For example, suppose that instead of a Gaussian, one takes the simple exponential form

$$g(x) = e^{-|x|/r_C} / (8\pi r_C^3) \quad , \quad (14a)$$

where the normalizing factors have been chosen to ensure that $\int d^3x g(x) = 1$. At first sight, one might think that such a choice would lead to the appearance of single inverse powers of r_C in various expressions, such as the small ℓ expansion of Γ_R and the stochastic heating rate (see Eqs. (9) and (7) respectively). This, however, is not the case, because as is made clear in Eq. (8.14) of the review of Bassi and Ghirardi [1], the function $g(x)$ enters the analysis only through the integral

$$G(y) = \int d^3x g(y-x)g(-x) \quad . \quad (14b)$$

When $g(x)$ is inversion invariant ($g(x) = g(-x)$), the order y term in the Taylor expansion of $g(y-x)$ vanishes on integration over x , and thus $G(y)/G(0)$ begins with an order y^2 term.

For example, the choice $g(x)$ of Eq. (14a) leads (with $s = |y|/r_C$) to

$$\begin{aligned} G(y) &= \frac{1}{64\pi} (1 + s + s^2/3) e^{-s} \\ &= \frac{1}{64\pi} [1 - s^2/6 + s^4/24 - s^5/45 + O(s^6)] \quad , \end{aligned} \tag{14c}$$

in which an odd power of $|y|$ appears first at fifth order! Hence with alternative choices of the correlation function, the qualitative form of Eqs. (9) and (7) remains unchanged, apart from a rescaling of r_C by a numerical constant. See Weber [1] for a further discussion of alternative choices of the correlation function.

Continuing, then, with the Gaussian choice for $g(x)$, let us consider the effect of changing r_C from the conventional, and rather arbitrary choice, of $r_C = 10^{-5}\text{cm}$. Clearly, if r_C were increased to $r_C \sim 10^{-4}\text{cm} = 1\mu$, the IGM heating rate given by Eq. (7) would be reduced by two orders of magnitude, and the upper bound on λ would be raised to $10^{10\pm1}$ times the standard CSL value.

We next analyze the effect on our latent image bounds of increasing r_C . We consider first the case of formation of a photographic latent image. For a small AgBr grain of diameter $\ell = 10^{-5}\text{cm}$, motion of the Ag and Br ions within the grain would be suppressed, according to Eq. (9), by a factor $\frac{1}{4}(\ell/r_C)^2 = 1/400$. However, since the bromine atoms that leave the grain typically move a distance of up to a micron into the gelatine, the suppression factor for these atoms would be only of order $\frac{1}{4} \int_0^1 dx x^2 = 1/12$. So with $n = 30 \times 80 = 2400$, and as before $N = 20$ and $\lambda = 2.2 \times 10^{-17}\text{s}^{-1}$, our revised estimate from Eq. (9) is $\Gamma_R = 2.2 \times 10^{-10}\text{s}^{-1}$, requiring λ to be $1.4 \times 10^{11\pm2}$ times larger than the standard CSL value.

Turning next to latent image formation in etched track detectors, when r_C is increased by a factor of 10, the value of n used in Eq. (9) also increases by a factor of 10, but N decreases by the same factor. Therefore the combination $n^2 N / r_C^2$ is a factor of 10 smaller

than estimated previously, and so an increase of λ by a factor of $\leq 5 \times 10^{10}$ is required for the reduction rate to equal the latent image formation rate, using attainment of chemical equilibrium as the criterion.

To conclude this section, we note that the physiology of the eye gives a possible argument suggesting that r_C should not be larger than 1 or 2 microns. Consider a dense array of photoreceptors, which one is trying to optimize *both* with respect to rapidity of response, which we assume to be given by the reduction rate Γ_R , and the ability to resolve fine detail. If the diameter of each photoreceptor is ℓ , there will clearly be a tradeoff between the two desired attributes of the array, as we vary ℓ . If ℓ is much smaller than r_C , the response rate of each detector element is suppressed by a factor of ℓ^4 , with a factor of ℓ^2 coming from the explicit ℓ^2 in Eq. (9), and an additional factor of ℓ^2 coming because the factor n in Eq. (9) scales as ℓ . (Since we are considering here the rate for a single photodetector element, the factor N in Eq. (9) will be 1). On the other hand, the spatial resolution of the array varies as ℓ^{-2} . Hence one gains more in response time than one loses in spatial resolution as ℓ is increased, until ℓ reaches r_C . Once ℓ is larger than r_C , the increase in the response time becomes only linear in ℓ rather than quartic (c.f. Eqs. (6a) and (6c), with n now a constant and N increasing in proportion to ℓ .) Thus increasing ℓ beyond r_C produces a gain in response time that is less than the corresponding loss of spatial resolving power. So these considerations suggest that an optimized photoreceptor array should have $\ell \sim r_C$.

Assuming that the evolution of the human eye has optimized it with respect to both response time and resolving power, and using the empirical fact [30] that the diameter of rods in the retina is 2μ , this reasoning suggests that the correlation length r_C is unlikely to be larger than of the order of a micron. However, some caveats are in order. First,

since physiological constraints prevent eucaryotic cells from being much smaller than of the order of microns, this argument cannot be used to disfavor smaller values of r_C , such as the conventionally assumed $r_C = 0.1$ micron. Also, in applying the optimized array argument to the rods of the eye, we have implicitly assumed that with the enhanced reduction rate suggested by latent image formation, reduction occurs directly within the individual rods, rather than requiring molecular motions in the optic nerve as well. As discussed briefly in Appendix C, estimates based on the known amplification chain in the rods support this assumption, but are incompatible with reduction occurring in the conformational change of an individual rhodopsin molecule. However, these estimates (if not overly optimistic) suggest such a high reduction rate in the rod amplification chain that reduction rate is no longer a relevant factor in determining the optimized array dimensions, which then undermines the argument just given relating the diameter of rods to the value of r_C .

7. Implications of a larger reduction rate

We turn now to a discussion of experimental implications of a greatly enhanced value of λ . We divide our analysis into two parts, first considering those experiments that do not rely on the secular increase of energy associated with the CSL process, and then considering experiments that depend on this secular energy increase (in which case the analysis possibly would be modified in dissipative extensions of the CSL model.) We consider two possible choices of enhanced CSL parameters, (I) $r_C = 10^{-5}\text{cm}$, $\lambda = 4 \times 10^{-10}\text{s}^{-1}$, and (II) $r_C = 10^{-4}\text{cm}$, $\lambda = 3 \times 10^{-8}\text{s}^{-1}$. These correspond respectively to the latent image lower bounds of 2×10^7 and 1.4×10^9 times the standard CSL value, and are near the lower end of the respective ranges permitted by the IGM heating upper bounds.

7A. Experiments not based on secular energy increase

Fullerene diffraction For case (I), with a grating of $10^{-5}\text{cm} = r_C$, setting $\Gamma_R = \lambda n^2 = 10^2\text{s}^{-1}$ gives a value of $n \sim 5 \times 10^5$ at which washing out of the diffraction pattern would set in. For case (II), with a grating of $\ell = 2.5 \times 10^{-5}\text{cm} = .25r_C$, setting $\Gamma_R = \lambda n^2 \ell^2 / (4r_C^2) = 10^2\text{s}^{-1}$ also gives a value of $n \sim 5 \times 10^5$ at which the diffraction pattern would start to wash out. Thus, diffraction experiments with projectiles of molecular weight of order 500,000 (a factor of 500 beyond what has been achieved so far) would confront the CSL model, when the parameter values are minimally enhanced to account for reduction in latent image formation.

Supercurrent decay For case (I), from the calculation of refs [17] and [18] we find a supercurrent decay rate of $10^{-19}\text{s}^{-1} \sim 1/(3 \times 10^{11}\text{years})$, while for case (II), we find a supercurrent decay rate of $8 \times 10^{-19}\text{s}^{-1} \sim 1/(4 \times 10^{10}\text{years})$. A direct measurement of the supercurrent decay time constant (as opposed to a time constant inferred from an improved resistivity measurement based on a short measurement time) would be needed to confront the CSL model with enhanced parameter values. Again, we note that these estimates ignore recombination processes in which Cooper pairs are created.

Mirror deflection experiment The papers of ref [2] give a detailed analysis of a mirror deflection experiment proposed by Marshall et al [31], as expressed in terms of the parameter η governing the small displacement CSL equation. For the geometry of the proposed experiment, with a cubical mirror of side S and density D , when $S \gg r_C$ the parameter η is given by the formula

$$\eta = 8\pi r_C^2 \lambda S^2 D^2 \quad . \quad (15)$$

When S is of order r_C , this formula has corrections of order unity, but we shall continue to

use it to give a rough order of magnitude estimate. For case (I) we find that η is a factor of 2×10^7 larger than in ref [2], giving a fringe visibility damping factor $e^{-\Lambda}$, with $\Lambda \sim 0.04$. The thermal decoherence background has been estimated recently by Bernád, Diósi and Geszti [32] as $\Lambda_T \sim 0.5$, so the case (I) CSL effect is not detectable. For case (II), we find that η is a factor of 1.5×10^{11} bigger than than in ref [2], corresponding to $\Lambda \sim 3 \times 10^2$, indicating complete suppression of the interference fringes, a large and detectable effect. Thus, even a version of the Marshall et al experiment that is a factor 100 times less sensitive than envisaged in their proposal would be of considerable interest, provided that thermal decoherence backgrounds can be kept small.

7B. Experiments utilizing secular energy increase

We discuss here experiments that utilize the fact that any non-dissipative Brownian process, including the CSL stochastic noise, will produce a secular increase in system energy and a corresponding increase in the rms values of translational and rotational displacements. Should it be possible to formulate a satisfactory dissipative version of CSL, the predictions for these experiments would be altered when the observational time exceeds a characteristic time for dissipative effects to set in (see ref [25]). Having stated this caveat, we proceed to give the results expected from the standard version of CSL, with enhanced parameters as suggested by latent image formation.

The Collett–Pearle rotational diffusion proposal Collett and Pearle [4] have proposed searching for the mean square rotational diffusion $\Delta\theta_{\text{CSL}}$ of a suspended disk of order r_C in dimensions. We write the result of their Eq. (C.6) in the form

$$\Delta\theta_{\text{CSL}} = \left(\frac{\hbar f_{\text{ROT}} I \lambda}{12} \right)^{\frac{1}{2}} \frac{t}{m_N r_C^2} \Delta\theta_{\text{SQL}} \quad , \quad (16a)$$

with f_{ROT} a dimensionless function of the ratios of disk dimensions to r_C given in ref [4], with I the disk moment of inertia, and with the “standard quantum limit” $\Delta\theta_{\text{SQL}}$ given by

$$\Delta\theta_{\text{SQL}} = \left(\frac{\hbar t}{I}\right)^{\frac{1}{2}}. \quad (16b)$$

For readers not familiar with standard quantum limits, Eq. (16b), and also the analogous result for the standard quantum limit on measurement of the translational coordinate of a free mass, are given in Appendix B. (Note that the moment of inertia I cancels when Eq. (16b) is substituted into Eq. (16a), which is why it does not appear in Eq. (C.6) of ref [4].) Following Collett and Pearle, we assume a disk radius $L = 2r_C$ and a disk thickness $b = 0.5r_C$, for which $f_{\text{ROT}} \simeq 1/3$, and the moment of inertia is given by $I = ML^2/4$, with M the disk mass. Assuming a disk density of 10^{24}cm^{-3} , we then find, for case (I), $\Delta\theta_{\text{CSL}}/\Delta\theta_{\text{SQL}} = 6.6 \times 10^2 t$, and for case (II) $\Delta\theta_{\text{CSL}}/\Delta\theta_{\text{SQL}} = 1.8 \times 10^4 t$, with t in seconds in both formulas. For estimates of the rotational diffusion produced by Brownian motion, which places severe pressure and temperature constraints on the environment of the disk, see Eq. (6.4) of ref [4]. Since the Brownian rotational diffusion scales as the product of the square root of the pressure and the fourth root of the temperature, increasing the CSL effect by a factor of order 10^3 to 10^4 makes the vacuum and cryogenic constraints on the experiment much less severe.

Nanomechanical oscillator We consider next the nanomechanical resonator reported by LaHaye et al [33], and discussed from the viewpoint of CSL effects in Adler [6], to which the reader is referred for details. Recalculating the numbers found in ref [6] to reflect an enhanced CSL parameter λ (and in case (II), an increased r_C), we find that the increase in the harmonic oscillator occupation number N over a time equal to the inverse of the noise

bandwidth is of order 10^{-5} in case (I), and of order 10^{-1} in case (II). Similarly, the root mean square deviation of the quantum nondemolition variables $X_{1,2}$ over a time equal to the inverse of the noise bandwidth is of order 3×10^{-3} of the corresponding standard quantum limit in case (I), and of order the 0.3 times the standard quantum limit in case (II). Hence the enhanced CSL effect would still be not accessible in case (I), and would come close to being detectable in case (II).

Gravitational wave detectors LIGO and LISA We consider finally gravitational wave detector experiments, also discussed from the viewpoint of CSL effects in ref [6]. The Advanced LIGO Interferometers [34] are expected to approach the standard quantum limit in position measurement accuracy, while the Laser Interferometer Space Antenna (LISA) [35] should achieve a position accuracy of around 10^4 times the standard quantum limit. Recalculating the numbers from ref [6], we find that for LIGO the root mean square stochastic deviation in the test mass coordinate over a time interval of 1/70 s will be ~ 0.02 times the standard quantum limit in case (I), and ~ 1.4 times the standard quantum limit in case (II). For LISA, the corresponding figures over a time interval of 10^4 s are $\sim 6 \times 10^4$ times the standard quantum limit (that is ~ 6 times the expected position accuracy) in case (I), and $\sim 5 \times 10^6$ times the standard quantum limit ($\sim 5 \times 10^2$ times the expected position accuracy) in case (II). Thus, the enhanced CSL parameter values give an effect that may be barely visible in LIGO in case (II), and should be readily visible in LISA in both cases (I) and (II).

The background from molecular collisions In designing experiments to look for CSL effects connected with the secular increase in energy and associated coordinate deviations, one will have to make sure that similar effects associated with molecular collisions are smaller

in magnitude. There are two ways of estimating such effects. The first is to use the decoherence calculation of Joos and Zeh [29] (with later corrections [29] in the numerical coefficient), which gives a rate parameter Λ for decrease in the off-diagonal density matrix element in a coordinate basis, that is analogous to the CSL parameter $\eta/2$. The second is to use standard classical Brownian motion formulas, as is done in the paper of Collett and Pearle [4]. In Adler [36] we calculate and compare the mean square coordinate fluctuation predicted by the two methods, for the case of isotropic hard sphere scattering. We show that the two calculations give identical results, and so either method can be used to estimate collision backgrounds that may mask CSL effects.

8. Summary and discussion

To summarize, we have found that if latent image formation constitutes measurement, then the parameters of the CSL model for objective state vector reduction must be much larger than conventionally assumed. The enhanced parameter values needed are compatible with empirical upper bounds, and suggest that CSL effects may be within reach of experimental detection within the next decade or two.

Further experiments on latent image formation would also be of great interest, to firm up (or falsify) assumptions that we have made in our analysis. It would be very useful to have more accurate information on the time needed for a latent image to form, in both photography and etched track detectors, since estimates of this time (or rate) were a crucial input for our analysis. Our central assumption, that latent image formation, and not subsequent development or etching, constitutes measurement, should also be subject to experimental test. In principle, a photographic emulsion or an etched track detector could

be used as a “which path” detector in one arm of a quantum interferometer; if latent image formation constitutes measurement, then the interference fringes should be destroyed by the presence of the emulsion or detector through formation of the latent image, which is a “fossilized record” of the path through the interferometer. According to the view expressed by Gisin and Percival [3] and adopted in this article, collapse onto a definite outcome should occur once the latent image has formed, and should not depend on a subsequent stage of amplification by development or etching.

Acknowledgments

I wish to thank E. Abrahams, A. Bassi, B. Barish, N. Brandt, D. Goldhaber-Gordon, P. Goldreich, J. Halliwell, L. Hui, E. Ippoliti, R. Joynt, A. Levine, J. Ostriker, N. Gnedin, L. Page, P. Pearle, G. Steigman, M. Tinkham, H. Robins, K. Thorne, and M. Turner for helpful conversations or email correspondence. This work was supported in part by the Department of Energy under Grant #DE-FG02-90ER40542. Part of this work was done while the author was at the Aspen Center for Physics.

Appendix A: Radiation by a free charge

We give here an alternative calculation of the radiation by a free charge, obtained by using the basic dipole radiation formula $P = \frac{1}{6\pi} e^2 a^2$, with $e^2/(4\pi) = 1/137.04$ and with a the acceleration of the particle of charge e . For a time average over the stochastic process, a^2 is to be interpreted as $E[(\ddot{x})^2]$, with $E[]$ the stochastic expectation. Rather than proceeding from the CSL equations, we use a modification of the CSL equations that has a unitary dynamics but leads to the same density matrix evolution; this substitution is allowed because the expectation $E[]$ of quantum transition probabilities is expressible directly in terms of the

density matrix. (This substitution is used by Fu [21] as the basis of his calculation.) The advantage of doing this is that the modified dynamics has a Heisenberg picture formulation with a self-adjoint Hamiltonian, which permits a direct calculation of \ddot{x} from the Heisenberg equations of motion.

Specifically, corresponding to the CSL model we have the effective coordinate representation Hamiltonian

$$H = \frac{p^2}{2m} - \frac{\hbar}{dt} \int d^3z (m/m_N) dB(z) g(z-x) \quad , \quad (A1)$$

from which we calculate

$$\begin{aligned} \dot{x} &= -\frac{i}{\hbar} [x, H] = \frac{p}{m} \quad , \\ \ddot{x} &= -\frac{i}{\hbar} [p/m, H] = \frac{\hbar}{m_N dt} \int d^3z dB(z) \partial_x g(z-x) \quad , \end{aligned} \quad (A2)$$

with ∂_x a vector gradient. Thus for $E[(\ddot{x})^2]$ we get

$$E[(\ddot{x})^2] = \frac{\hbar^2 \gamma}{m_N^2 dt} \int d^3z [\partial_x g(z-x)]^2 = \frac{3\hbar^2 \lambda}{2m_N^2 r_c^2 dt} \quad . \quad (A3)$$

Using $1/dt = \delta(0) = (2\pi)^{-1} \int_{-\infty}^{\infty} dk = \pi^{-1} \int_0^{\infty} dk$, and substituting into the dipole radiation formula, we get

$$P = \frac{e^2}{4\pi^2} \frac{\hbar^2 \lambda}{m_N^2 r_c^2} \int_0^{\infty} dk \quad , \quad (A4)$$

in agreement with Eq. (11) when specialized to units with $\hbar = 1$. The total radiated power is infinite, but since CSL is a nonrelativistic theory, the calculation is reliable only for $k \ll m$. With a cutoff taken at $k = m$, the radiated power is smaller than the power gain through stochastic heating (see Eq. (7)) by a factor $e^2/(3\pi^2)$.

We also note that exactly the same result is obtained from the so called QMUPL model, which corresponds to the leading small-displacement Taylor expansion of the CSL

model (and of the original Ghirardi–Rimini–Weber [37] model as well), as discussed and further referenced in refs [2]. In this case the effective Hamiltonian is

$$H = \frac{p^2}{2m} - \frac{\hbar}{dt}(m/m_N)\sqrt{\eta}dBx \quad , \quad (A5)$$

with $(dB)^2 = dt$ and $\eta = \lambda/(2r_C^2)$, and one immediately finds

$$\ddot{x} = \frac{\hbar}{m_N} \frac{\sqrt{\eta}}{dt} dB \quad . \quad (A6)$$

Since the QMUPL model of Eq. (A5) is one-dimensional, one must multiply by a factor of 3 in forming the expectation of the squared acceleration from Eq. (A6), giving the same result as obtained in Eq. (A3) from the CSL calculation. As one would expect, the integral over the squared gradient of $g(z-x)$ in Eq. (A3) is just the one encountered in evaluating η from the second order expansion of the function $G(y)$ of Eq. (14b), in the course of deriving the QMUPL model from the small displacement expansion of the CSL model.

Aside from giving a simpler derivation of Fu’s result (which he obtained by using Feynman rules to calculate the S matrix for the radiation process), the derivation given here shows that the radiation is directly attributable to the noise-induced acceleration. Moreover, the radiation has a coefficient that is independent of the particle mass m , because the $1/m$ in the relation between \dot{x} and p cancels against the m coming from the noise coefficient m/m_N included in the mass-proportional coupling scheme. This cancellation of m dependence is why the radiation field from the germanium core largely cancels that from the valence electrons.

Appendix B: Standard quantum limits

We derive here the standard quantum limit for measurement of the angle θ through which a suspended disk rotates about the axis of suspension, taken here as the z axis. We

then state by analogy the corresponding standard quantum limit for the measurement of a translational coordinate.

Let $L_z = -i\hbar\partial/\partial\theta$ be the component of angular momentum around the z axis, which obeys the commutation relation

$$[\theta, L_z] = i\hbar \quad . \quad (B1)$$

We shall only consider very small angular displacements, and so we neglect complications associated with the 2π periodicity of θ and the corresponding integer quantization of L_z . From the commutation relation of Eq. (B1), one gets an uncertainty relation of the usual form,

$$\Delta\theta\Delta L_z \geq \frac{\hbar}{2} \quad . \quad (B2)$$

Suppose at $t = 0$ a measurement of θ is made to accuracy $\Delta\theta$. This induces an uncertainty $\Delta L_z \geq \hbar/(2\Delta\theta)$, and since $\Delta L_z = I\Delta\omega_z$, with I the moment of inertia of the disk around the z axis and ω_z the corresponding angular velocity, the squared uncertainty in the angular coordinate of the disk at time t is

$$(\Delta\theta(t))^2 = (\Delta\theta)^2 + (t\Delta\omega_z)^2 = (\Delta\theta)^2 + (t\hbar)^2/(2I\Delta\theta)^2 \quad . \quad (B3)$$

Minimizing this expression with respect to $\Delta\theta$, we get the standard quantum limit on the measurement accuracy of θ at time t ,

$$(\Delta\theta(t))^2 \geq (\Delta\theta_{\text{SQL}})^2, \quad \Delta\theta_{\text{SQL}} = (\hbar t/I)^{1/2} \quad . \quad (B4)$$

In an entirely analogous fashion, from the commutation relation $[x, p] = i\hbar$, together with $p = Mv$ with M the mass and v the velocity, one derives the standard quantum limit

for measurement of a translational coordinate, $\Delta x_{\text{SQL}} = (\hbar t/M)^{1/2}$. For further details of standard quantum limits, see ref [38].

Appendix C: Where does reduction occur in the visual system?

We give here some estimates for the implications of an enhanced λ parameter for the human visual system. Attention was first drawn to the issue of possible state vector reduction in the visual system by Albert and Vaidman [39], with a response in Aicardi et al [39], as reviewed in Bassi and Ghirardi [1]. An issue raised in this early work is: where does one expect reduction to occur in the visual system – at the retina, or higher up in the nervous system or brain?

To address this question, we shall assume case (I) discussed in Sec. 7, that is $r_C = 10^{-5}\text{cm}$ and $\lambda = 4 \times 10^{-10}\text{s}^{-1}$. We first ask whether the cis-trans conformational change of an individual rhodopsin molecule on absorption of a photon can give reduction with these parameter values. A rhodopsin molecule has a molecular weight of about 4×10^4 nucleons, and a diameter of about $4 \times 10^{-7}\text{cm}$. Assuming that in the conformational change all of the nucleons in the molecule move by a molecular diameter (likely a considerable overestimate) we find from Eq. (9), with $n = 4 \times 10^4$, $N = 1$, and $\ell/(2r_C) = 2 \times 10^{-2}$, a reduction rate of $\sim 3 \times 10^{-4}\text{s}^{-1}$. In other words, reduction with these parameters requires of order $3 \times 10^3\text{s}$, while the conformational change occurs in $200\text{fs} = 2 \times 10^{-13}\text{s}$, so even with the enhanced λ parameter, reduction cannot occur during the conformational change. For a detailed discussion of the possibility of reduction during conformational change, and many useful references, see Thaheld [40].

On the other hand, with the enhanced λ parameter, there is no apparent difficulty

in reduction occurring during the amplification chain in a rod cell, without invoking signal transport in the nervous system to which the rod is linked. According to the review of Rieke and Baylor [41], a single catalytically active rhodopsin leads to the closure of several hundred ion channels, which over the response time of 300ms blocks the entrance of about 3000 cations (sodium or potassium) to the outer segment of the rhodopsin molecule. Assuming that the relevant ℓ here is of the order of the rod cell diameter of 2 microns $> r_C$, we can use the estimate of Eq. (8), with $N = 1$ and $n = 300 \times 3000 \times 23 \sim 2 \times 10^7$, giving a reduction rate of $2 \times 10^5 \text{s}^{-1}$. Even if our assumptions here about n and ℓ are optimistic, this estimate indicates that with the enhanced λ parameter, there is plenty of latitude for reduction to be complete by the end of the amplification chain in the individual rod cell. Note that with the original CSL λ parameter value, the reduction rate would be too small by about three orders of magnitude for reduction to occur in a rod cell, requiring the invocation of signal transport in the nervous system, as discussed in ref [1].

Appendix D: Reduction arising from phonon emission and thermal fluctuations

In this appendix we estimate the reduction rate associated with phonon emission by electrons and ions, and also the reduction rate associated with thermal lattice fluctuations.

To start, we note that phonons of wavelength shorter than r_C lead to no net center of mass displacement of a block of emulsion of linear dimensions r_C , and so are not the main phonon contributors to reduction. On the other hand, long wavelength phonons can lead to a displacement of the center of mass of such a block, and so can potentially have a significant effect. Thus, we are interested in making an estimate of the reduction effect of the emission of long wavelength acoustic phonons. The effect of short wavelength optical phonons will be

taken into account afterwards when we estimate the reduction rate associated with thermal lattice fluctuations.

Consider, then, a lattice block of total mass M with atom site coordinates u_i . For a phonon of angular frequency ω , each site will oscillate as $u_i = a \cos(\omega t + \delta_i)$, with δ_i a site-dependent relative phase, and so the time-averaged velocity is $\langle u_i^2 \rangle = a^2 \omega^2 / 2 \equiv \omega^2 \ell^2$, independent of i . Since the phonon energy is half kinetic and half potential, we thus have

$$\frac{1}{2} M \omega^2 \ell^2 = \frac{1}{2} \hbar \omega \quad , \quad (D1a)$$

giving for the mean square atomic displacement ℓ^2 ,

$$\ell^2 = \frac{\hbar}{M \omega} \quad . \quad (D1b)$$

For a block composed of N sub-blocks of dimension of the correlation length r_C , each containing n nucleons, we have $M = N n m_N$, while the reduction rate is given by Eq. (9) in terms of N, n, ℓ . Substituting Eq. (D1b) we get $\Gamma_R = \lambda f(\omega)$, with

$$f(\omega) = \frac{\hbar n}{4 r_C^2 m_N \omega} \quad . \quad (D2a)$$

Note that the factor N has dropped out, because when the lattice contains many coherent length sized blocks, the N in Eq. (9) cancels against the $1/N$ implicit in Eq. (D1b), which reflects the fact that the energy of the single phonon is dispersed over the whole lattice, and so the mean squared amplitude of oscillation of each atomic site scales as $1/N$. Note also that Eq. (D2a) still assumes that the phonon wavelength is longer than r_C , so that all nucleons within a block of size r_C move together, and the n^2 factor in Eq. (9) is applicable. To account for the loss of coherence when the phonon wavelength is shorter than r_C , a factor

$$G(\omega) = \min[1, (\lambda/r_C)^3] = \min[1, (2\pi c_s/(r_C \omega))^3] \quad (D2b)$$

must be included in Eq. (D2a), with c_s the acoustic phonon velocity (the velocity of sound, typically $c_s \sim 3 \times 10^5 \text{cm s}^{-1}$). This factor takes the value of unity for $\lambda \geq r_C$, and approaches $1/n$ as λ approaches the lattice spacing $\sim 10^{-8} \text{cm}$. We will proceed by first doing our estimates without including G , and then noting the correction factor, denoted by $\langle G \rangle$, when G of Eq. (D2b) is included in the integrands.

Consider now an electron with initial wave number k_0 that slows down through multiple emissions of acoustic phonons, as described in Kittel [42]. We wish to integrate Eq. (D2a) over the spectrum of emitted phonons, to get the total reduction rate λf_{tot} attributed to the emitted phonons. Letting $R(k(t), q)$ be the rate of production of phonons of wave number q when the electron wave number is $k(t)$, we have

$$f_{\text{tot}} = \int_0^T dt \int_0^{q_{\text{max}}} dq R(k(t), q) f(\omega(q)) \quad . \quad (D3a)$$

The upper limit T of the time integration is determined by $k(T) = k_{\text{th}}$, where $k_{\text{th}} = (3m^*k_B T)^{1/2}/\hbar$ is mean thermal electron wave number (with m^* the electron effective mass), since once the electron has been thermalized by phonon emission, it is in an equilibrium where phonon absorption by the electron is as important as phonon emission. We shall assume that both k_0 and k_{th} are much larger than the threshold momentum for emitting acoustic phonons, given in terms of m^* and c_s by $k_{\text{min}} = m^*c_s/\hbar$. With this assumption (which we shall verify), the upper limit q_{max} giving the maximum phonon wave number that can be emitted by an electron of wave number k is given by $q_{\text{max}} = 2k$. Also with this assumption, the time evolution of the electron wave number k is given by $dk/dt = -\sigma k^3$, with σ a constant that will end up dropping out of the calculation. This allows us to change

variables from t to k , and so Eq. (D3a) becomes

$$f_{\text{tot}} = \int_{k_{\text{th}}}^{k_0} dk \sigma^{-1} k^{-3} \int_0^{2k} dq R(k, q) f(c_s q) \quad , \quad (D3b)$$

where in the argument of f we have substituted $\omega = c_s q$.

The analysis of Kittel [42] also gives an expression for $R(k, q)$,

$$R(k, q) = \frac{5}{16} \sigma \frac{q^2}{k} \quad , \quad (D4)$$

with σ the same constant that appears in the time evolution of the electron wave number.

So substituting Eq. (D4) into Eq. (D3b), and doing the integrals, we get

$$f_{\text{tot}} = \frac{5}{32} \frac{\hbar n}{r_C^2 m_N c_s} \left(\frac{1}{k_{\text{th}}} - \frac{1}{k_0} \right) \simeq \frac{5}{32} \frac{\hbar n}{r_C^2 m_N c_s k_{\text{th}}} \quad ; \quad (D5a)$$

note that this result is essentially independent of the initial electron wave number k_0 , as long as this is significantly greater than the thermal wave number k_{th} . Taking the electron effective mass m^* equal to the electron mass, we get (at room temperature) $k_{\text{th}} = 10^7 \text{cm}^{-1}$, while for the minimum wave number for phonon emission we get $k_{\text{min}} = 0.3 \times 10^6 \text{cm}^{-1}$, which is much smaller than k_{th} , as assumed. Evaluating Eq. (D5a) with this value of k_{th} , with $r_C = 10^{-5} \text{cm}$ and $n = 10^9$, we get $f_{\text{tot}} \simeq 0.3 \times 10^3$, giving $\Gamma_R = \lambda f_{\text{tot}} \sim 10^{-14} \text{s}^{-1}$, which is a factor of 10^6 smaller than the latent image formation estimate of Eq. (8). This is still an overestimate, since we have not included the factor G of Eq. (D2b); when the integrals are redone with the factor of G included, we find an additional factor in Eq. (D5a) of

$$\langle G \rangle = \left(\frac{\pi}{r_C k_{\text{th}}} \right)^2 \simeq 10^{-3} \quad . \quad (D5b)$$

Thus the reduction rate induced by phonon emission from an electron emitted from a silver halide molecule is more than a factor of 10^9 smaller than Eq. (8).

The silver and bromine ions that diffuse to the surface of the grain will already be in thermal equilibrium. We assume that they obey a dispersion relation of the form $E = p^2/(2m^*)$, with m^* an effective mass similar to the ionic mass, and so their mean wave numbers will be given in terms of this effective mass by $k_{\text{th}} = (3m^*k_B T)^{1/2}/\hbar$. For a particle of effective mass m^* the ratio $k_{\text{min}}/k_{\text{th}}$ scales as $(m^*)^{1/2}$, and so for an ion of atomic weight ~ 100 we get by comparison with the electron case calculated above,

$$k_{\text{min}}/k_{\text{th}} \sim (2 \times 10^5)^{1/2} \times 3 \times 10^{-2} \sim 10 \quad . \quad (D6)$$

Hence only the extreme tail of the thermal velocity distribution for these ions is above threshold for phonon production, leading to a suppression of the associated reduction rate by a factor $\exp(-1.5k_{\text{min}}^2/k_{\text{th}}^2) \sim 10^{-65}$.

When both electrons and interstitial ions are in thermal equilibrium, they continue to emit and absorb phonons. The associated reduction rate will be driven by the thermal average of $\sum_i (u_i^1 - u_i^2)^2$, with $u_i^{1,2}$ the atomic coordinates in superimposed states in the wave function labeled by the respective indices 1,2. An upper bound on this sum should be given by ignoring expected cancellations in the individual terms (that is, for most atoms we expect $u_i^1 \simeq u_i^2$), and replacing the sum by twice the thermal average of $\sum_i u_i^2$. The associated reduction rate is given in turn by the twice the integral of Eq. (D2a), including the factor G of Eq. (D2b), over the Debye phonon spectrum. When $G = 1$, the result is $\Gamma_R = 2\lambda f_{\text{tot}}$, where

$$f_{\text{tot}} = \frac{9}{4} \frac{k_B T n^2 N}{r_C^2 M_{\text{atom}} \omega_D^2} \quad , \quad (D7a)$$

with ω_D the Debye frequency. [A similar result, with ω_D^2 replaced by $3\omega_{\text{opt}}^2$, with ω_{opt} the lattice vibration frequency, is readily obtained from Eq. (9), when ℓ^2 is taken as the thermal

average of the atomic position, given by $\langle u^2 \rangle = 3k_B T / (M_{\text{atom}} \omega_{\text{opt}}^2)$.] Including the coherence factor G in the integral over the Debye spectrum gives an extra factor

$$\langle G \rangle = 3\pi c_s / (r_c \omega_D) \quad . \quad (D7b)$$

Evaluating Eqs. (D7a) and (D7b) at room temperature, with $r_C = 10^{-5}\text{cm}$, $n = 10^9$, $M_{\text{atom}} = 100m_N$, and $\omega_D = 3 \times 10^{13}\text{s}^{-1}$, we get

$$\Gamma_R \sim 2 \times 10^{-9} N \text{s}^{-1} \quad , \quad (D8a)$$

or with $N = 20$,

$$\Gamma_R \sim 4 \times 10^{-8} \text{s}^{-1} \quad , \quad (D8b)$$

a factor of three bigger than the latent image formation estimate of Eq. (8). As noted above, Eq. (D8b) is expected to give a substantial overestimate of the contribution of thermal fluctuations to the reduction rate, since for most atoms one expects $(u_i^1 - u_i^2)^2 \ll (u_i^1)^2 + (u_i^2)^2$.

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